Analytic Modeling of Insurgencies

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Abstract

Legacy force-on-force models are symmetrical; they reflect the fact that the dynamics of both sides in the conflict is governed by similar rules. These models have also traditionally focused on the physical aspects of armed conflicts – fire, attrition, detection, movement, logistics, etc. Insurgency models are different in two key aspects: (a) they are asymmetrical and (b) they contain, in addition to the physical aspects, significant cognitive, social, cultural, political and economic factors. The reason for the asymmetry is the way insurgents evolve and operate, and the emergence of non-physical factors is due to the major role of civilians in such conflicts. This article focuses on analytic modeling of insurgencies and counterinsurgency operations, describing both the physical and non-physical components of such armed conflicts.

Key words: Insurgency, Counterinsurgency, Situational Awareness, Civilians, Lanchester

1. Introduction

Combat modeling is one of the oldest areas of operations research, dating back to the Kriegsspiels—board war-games developed in the early 19th century for training, planning, and testing military operations in the Prussian Army. The ground-breaking work of Lanchester in 1916 [1] marks the beginning of formal models of conflicts, where mathematical formulas and, later on, computer simulations replace board games and sand tables. From WWII until a decade ago combat models have mostly been focused on the physical aspect of conflicts. These models have been typically applied to regular force-on-force engagements, where adversary armies, navies and air-forces engage in, so called, “kinetic” actions that comprise fire, maneuver and attrition.

Insurgencies are not new either; they date as far back as the Jewish revolt against the Seleucid Empire in the second century BC. Since then, insurgencies have been prevalent throughout history. While fighting the Japanese Imperial Army occupying China, Mao Tse-tung was one of

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14. ABSTRACT
Legacy force-on-force models are symmetrical; they reflect the fact that the dynamics of both sides in the conflict is governed by similar rules. These models have also traditionally focused on the physical aspects of armed conflicts ? fire, attrition, detection, movement, logistics, etc. Insurgency models are different in two key aspects: (a) they are asymmetrical and (b) they contain, in addition to the physical aspects, significant cognitive, social, cultural, political and economic factors. The reason for the asymmetry is the way insurgents evolve and operate, and the emergence of non-physical factors is due to the major role of civilians in such conflicts. This article focuses on analytic modeling of insurgencies and counterinsurgency operations, describing both the physical and non-physical components of such armed conflicts.

15. SUBJECT TERMS

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the first to offer a conceptual "model" of insurgencies in 1937 [2]. He described the insurgents as "fish" that must swim in the population's "sea" to survive and prevail. More recent and formal models of insurgencies include Deitchman's Guerrilla Warfare model [3], which is a Lanchester-based mathematical model (see more details about this model later on), and McCormick's Magic Diamond model [4], which is a conceptual model that identifies the players in an insurgency and specifies the interrelationships among them. However, it is only since the early 2000s, following major insurgencies in Colombia, Iraq, Afghanistan, Libya and elsewhere in the Middle East, that operations-oriented modeling of these armed conflicts has become prevalent. In particular, during the Iraq and Afghanistan wars operations-research analysts were deployed with combat units to collect and analyze data. They also utilized reach-back support from operations research analysts in the US for developing decision support models for both combat tactics against the insurgents and information operations aimed at gaining the support of the local population [5].

Insurgencies are different than regular force-on-force engagements. The two adversaries in an insurgency are *government* organized forces on one side, and, on the other side, loosely organized non-state actors, called henceforth *insurgents,* such as violent demonstrators, armed rebels and terrorists. While there are some differences between *Counterinsurgency* and *counterterror* operations – mostly in terms of scale of operations and countermeasures (see, e.g., [6]) – in some places in the following the two terms may be used interchangeably.

In one-on-one situations, the non-state actors – the insurgents – are no match to the state-controlled government forces, who are significantly larger, better equipped and better trained. To avoid eradication, the insurgents must reduce their signature as targets, and this elusiveness is attained by blending in with the civilian population among which the insurgents operate. The insurgents use relatively simple, yet lethal, weapons such as small arms, improvised explosive devices, and suicide bombs.

The asymmetry described above is one significant characteristic of insurgencies. The second characteristic is the active role played by civilians who provide insurgents, either willingly or as a result of coercion, hiding places, shelters, logistical support, and most importantly – information and recruits. Civilians play other roles too: they may provide information (intelligence) to the government forces regarding insurgents' activities and whereabouts, they consume social and economic resources that are provided either by the insurgents or the government, and they are possible targets to violent actions by both sides (see e.g., the conflict in Syria that started in 2011). All these characteristics make civilians a key component in insurgency modeling, which is absent in legacy armed-conflict models. In particular, models from behavioral science, sociology, political science and economics play a major role in insurgency modeling.

There are several ways to model insurgencies. Probably the most popular models are detailed (e.g., agent-based) simulations that represent both physical and cognitive interrelations among stakeholders in such conflicts. Some examples of simulation models are mentioned later on.

Another possible modeling approach is system dynamics [7]. A “softer” side of insurgency analysis – by means of political science, sociology and behavioral science – is embodied in the works of Kilcullen [8], which has been influential in planning counterinsurgency operations in Iraq and Afghanistan. Another important political and economic analysis of an insurgency is reported in [9].

In this article, however, we focus on OR analytic modeling based on formal methodologies such as differential equations, utility theory, game theory, and probability models. Besides their elegance, the conceptual simplicity of these models provides transparency, facilitates a clear description of cause-and-effect relations, and thus offers strategic insights regarding insurgency situations. Because of these features, some of the models presented below have been briefed to top leadership in the US Army.

Following a general discussion on analytic modeling of insurgencies presented in the next section, we describe three families of models addressing three major issues: (1) *Eyes and Ears*: the impact of information, intelligence and situational awareness on the conduct of counterinsurgency operations, (2) *Hearts and Minds*: the effect of civilians’ behavior and attitude towards the insurgents and the government, and (3) *Bullets and Fire*: the “kinetics” of insurgencies that represents the physical attrition in this type of conflicts. The three families are obviously related – e.g., informational models may have some attrition and behavioral components. Thus, the classification mentioned above – information, behavior and attrition – only serves for highlighting the main thrust of the models included in each corresponding section.

2. The Big Picture

The three major players in an insurgency are the two adversaries – the government forces and the insurgents – and the civilian population, which is caught in the middle. A fourth player is the international community [4], [10] who may (e.g., Libya, 2011) or may not (e.g., Syria, 2011-2014) be actively involved in the conflict. Because international forces, if they get actively involved in the insurgency, typically side with either the government (e.g., Afghanistan) or the insurgents (e.g., Libya), we will focus only on the three main players, as shown in Figure 1.

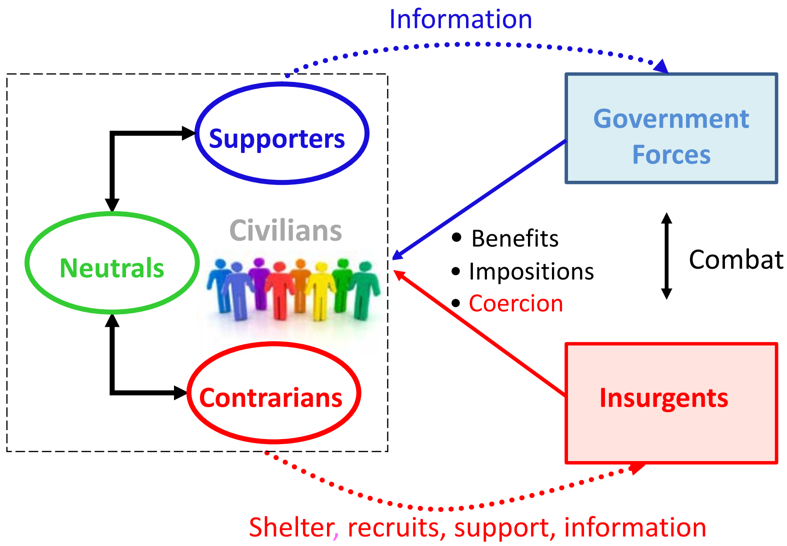

Figure 1: The Stakeholders in an Insurgency

The government and the insurgents are engaged in an armed conflict (the “Combat” link in Figure 1) governed by violent actions and mutual attrition. The civilian population is divided into three groups: *supporters* of the government, supporters of the insurgents called henceforth *contrarians* and those who maintain neutrality. The government and the insurgents, while trying to gain the support of the population, provide social benefits such as healthcare and education, but they may also impose some requirements such as dress codes, taxes and draft. The insurgents also use aimed violence against civilians to coerce them to collaborate and deter them from supporting the government. These actions, combined with observations on the state of the armed-conflict between the insurgents and the government forces, lead to shifts in civilians’ behavior regarding their support to the two sides. Contrarians support the insurgents by providing hiding places, recruits, logistics support and information, while government supporters mostly provide information, in the form of human intelligence (HUMINT), to government forces.

As mentioned earlier, the blue block corresponding to Government Forces in Figure 1 may contain several stakeholders such as regular government army, police forces, and international forces supporting the government. The red block corresponding to the Insurgents may also comprise non-homogeneous forces in the form of a number of competing tribal or sectarian militias who may also fight each other to gain control of the insurgency (e.g., Syria 2013-2014).

There were several attempts to develop analytic insights regarding the observed evolution of insurgencies. Bohorquez and others [11] examine the patterns of violence in insurgencies and terror events and identify a common pattern regarding the size distribution of such events and their timing. Their dynamic model explaining this pattern is based on the notion of coalescence and fragmentation of insurgents or terror organizations, thus producing ecology of groups. A more recent paper [12] reveals a dynamical pattern of fatal insurgency attacks. This pattern, which is manifested in a power law, identifies possible escalation scenarios of such attacks. The authors establish a new metric for understanding the momentum of these attacks and the effectiveness of COIN actions—a metric that appears to be stable across multiple conflicts and at different scales.

3. Eyes and Ears

Situational awareness regarding the battlefield, generated by a steady, relevant and reliable stream of intelligence reports, is an important factor in any armed conflict. In order to effectively deploy and operate its weapons and other combat assets, a military force needs to know the deployment, capabilities, plans and intentions of its adversary, as well as details regarding the combat environment. Situational awareness becomes even more critical in counterinsurgency operations because of the two aforementioned characteristics: asymmetry and the significant active role of civilians. First, the asymmetry is manifested in elusive, well hidden insurgents, diffused among civilians and thus reducing their signature as targets. This elusiveness requires extra effort by the government forces – mostly by acquiring intelligence from human sources – to locate and effectively engage the insurgents. Second, the impact of civilians on the evolution of an insurgency necessitates social, cultural and behavioral intelligence about the population's attitude, mood and sentiments. This type of intelligence is typically absent in legacy force-on-force conflicts (see also next section).

3.1 Generalized Deitchman Model

The first to capture the asymmetry feature in a Lanchesterian setting was Deitchman [3] in his Guerrilla Warfare model, which is a pair of differential equations. If $G(t)$ and $I(t)$ are the sizes of the government forces and the insurgents at time *t*, respectively, and *P* is the size of the civilian population, then Deitchman's model states that

$$G'(t) = -\alpha I(t)$$
$$I'(t) = -\gamma G(t)\frac{I(t)}{P}$$

where α, γ are attrition coefficients and the signature of the insurgents – their proportion in the population – is represented by $\frac{I(t)}{P}$. The larger the population among which the insurgents are diffused, the smaller is the insurgents' signature and thus the smaller the effectiveness of the government forces in fighting the insurgents. Deitchman's model was extended in [13], [14] and [15]. Absent accurate situational awareness, that is, information regarding the whereabouts of the insurgency targets, not only might the guerrillas be able to continue their insurgency actions unhindered, but collateral damage caused to civilians from poor targeting by the government forces may generate an adverse response against the government, thus creating popular support for the insurgents [16]. This popular support may translate into new cadres of recruits to the insurgency ranks [17]. This dynamics of two-sided attrition, collateral casualties among civilians, recruitment to the insurgency and reinforcement to the government forces, is captured by a pair of differential equations in [14]. The imperfect situational awareness, which results in collateral casualties, turns out to be crucial. Analyzing the differential-equations model results in some operational insights. While perfect targeting would eradicate the insurgents in no time, it is shown that, under reasonable assumptions, the government forces can never do that if the situational awareness is imperfect. The best the government forces can hope for is containing the insurgency at some manageable size, a result that has been confirmed empirically in recent insurgencies (e.g., Afghanistan, Syria). Moreover, it is shown that there may be two steady-state containment situations. One stalemate scenario involves relatively low level of violent activity by the government forces. However, this stalemate is fragile from the government point of view; small changes in the balance of forces could lead to quick government demise. The other steady-state containment scenario involves high level of violent activity and is more stable. In any case, the only alternative, which is the worse-case scenario for the government, is when the latter actually loses, as was the case in Libya, Tunisia and Yemen in 2011 (aka “Arab Spring”).

3.2 Searching for Insurgents

Optimizing the search for hiding insurgents could be modeled as a *whereabouts search* problem [18]. The objective here is to detect, as fast as possible, an insurgents’ cell that is hidden in one out of n possible locations observed by an imperfect sensor. The cost of searching a location is c, and the intelligence obtained from this search may be false negative (the searcher failed to detect an insurgents’ cell) with probability $1 – p$, false positive (the searcher erroneously identifies a location as an insurgents’ cell), with probability q. An indication by the sensor that a certain location is an insurgents’ cell, may be *correct* – insurgents are indeed hiding in that location – or wrong – the location is void of insurgents. Following such an indication (correct or wrong), a combat team is sent out with the objective to capture the insurgents in the indicated location. The operational costs of such actions are C_P and C_W, for *correct* and *wrong* scenarios, respectively. In case the indication is wrong, the cost may be collateral damage and lost of time and effort. It is shown in [19] that the optimal search sequence of the sensor follows a greedy policy that only depends on the prior location probabilities and the values of $\dfrac{p}{C_P + qC_W}$ in the various locations.

3.3 Balancing Between Intelligence Collection and Analysis

Collecting, processing and analyzing intelligence for obtaining a current and reliable situational awareness is a complex task that requires considerable resources. In particular, it is crucially important to balance off between the collection effort – gathering information and data from an assortment of sources – and the analysis effort – analyzing the collected information and producing useful operational knowledge. In a model that addresses this issue [20], using game-theoretic arguments in a queuing setting, assuming two types of traffic – light and heavy, the authors identify equilibria under various operational scenarios. They conclude that, contrary to current practices in the counterterrorism intelligence community, more should be invested in analysis so as to reduce the size of the queue of intelligence input.

4. Hearts and Minds

First coined by the British in Malaya during the early 50’s of the last century, and later on used in Vietnam (60’s) and more recently in Iraq and Afghanistan, “winning hearts and minds” is an operational concept that aims at swaying public support towards one side in the conflict. Because public opinion plays a major role in shaping the way an insurgency evolves, both sides – the government and the insurgents – take actions to win the support of the people. Which actions are effective? How do these actions interact? What is the end-effect of these actions? These types of questions have been addressed mostly in simulations [21], such as agent-based simulations [22], [23]. Various empirical approaches for addressing these questions have been reported in the political science literature, e.g., [9], [24]. Modeling public behavior takes into account the difference between the *attitude* of an individual (his “heart”) and his *behavior* (his “mind”). For example, a person who fundamentally opposes the insurgents in his heart (attitude) may express latent or even active support to the insurgents in his mind (behavior) based on pragmatic considerations. The terms attitude and behavior used here are related, respectively, to the terms “private preference" and “public preference" used in [25]. While it may be hard to change the attitude of people – their fundamental beliefs and values that have been shaped over centuries of cultural evolution – it may be possible to affect their manifested behavior, which is influenced by interests and utilities.

4.1 Carrots and Sticks

An analytic model that captures the aforementioned utilitarian aspect is presented in [26]. The situation modeled therein is as follows: a certain region is under the control of the insurgents who act there quite freely. Civilians in that region react to these actions based on their perception regarding the capabilities and intentions of the government, had it been in control instead of the insurgents, to improve or worsen their welfare. The insurgents execute two types of actions: (a) violent actions, aimed to coerce potential supporters of the government, which are only targeted at such suspected civilians (y), and (b) non-violent supportive economic and social actions aimed at gaining the population support (x). The dilemma of the insurgents is how to balance these two types of actions – the “sticks” y and the “carrots” x. A dynamic utility-based model is developed in [26] in which the state variables are the fractions of contrarians (supporters of the insurgency) (C), latent supporters of the government (L) and active supporters of the government (A) in the population. The model is parameterized by the activity levels of the insurgents (x,y) for both non-violent and violent actions, which are assumed to be bounded by the available resources to the insurgents, as depicted in Figure 3. It is shown that the model converges into equilibrium, and the main insight is that tipping points may occur, where small changes in the aforementioned balance between insurgents' violent (y) and non-violent (x) actions can drastically change the size of active supporters who help the government. See Figure 2.

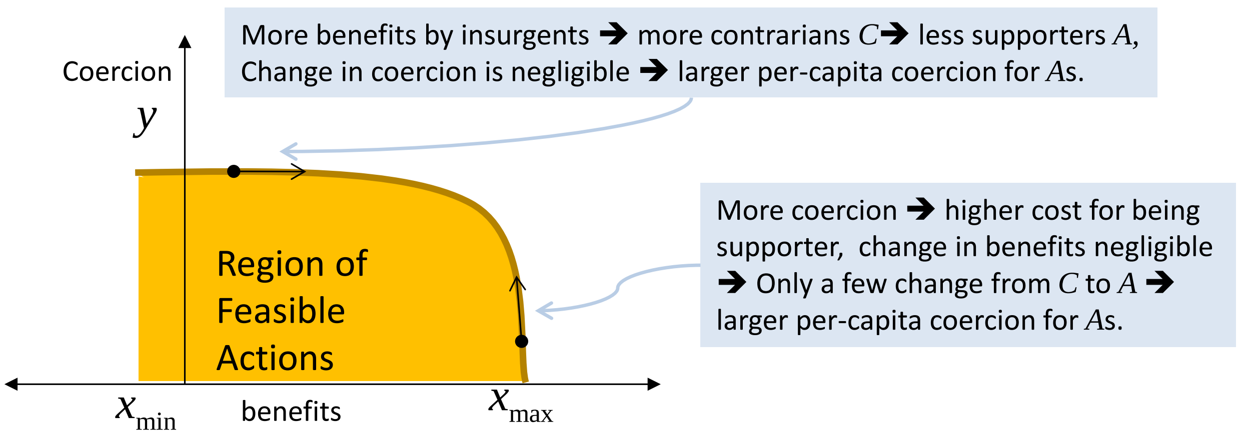

Figure 2: Cascading Effects of Insurgency's Actions

The government should be aware of potential tipping points that lead to the elimination of most active supporters and attempt to avoid situations that lead to them.

4.2 Civilians' Reaction to Violence

How civilians remember and react to violent actions by both sides – the insurgents and the government – is modeled in [27]. Arguably, the exposure to violence affects the people's sense of security and is a major factor in shaping their allegiance to one side or another. Ceteris Paribus, people will support the side that provides a better sense of security [17] [28]. A key question is how do people remember violence; is it the first exposure to violence that shapes their behavior or the last such encounter? The dynamic differential-equations model in [27] takes into account the violence intensity ratio of the government and the insurgents, the effectiveness of coercion by the insurgents, the targeting accuracy (see Section 3 above), and the way civilians remember and respond to violence. The main conclusions of the analysis are: (1) Excess violence and poor targeting accuracy may lead to situations where civilians' support for a certain side will vanish; (2) The government should not be discouraged by an initial small level of popular support because there are situations where this would actually play to its advantage if the insurgents are very violent and have poor situational awareness; (3) The effect of the initial distribution of opinions (support or opposition to a certain side) among civilians on the outcome of the insurgency depends on the way people remember and respond to violent experiences. For some responses the outcome is insensitive to this initial distribution.

5. Bullets and Fire

As observed by Deitchman [3] and others (see Section 3 above) counterinsurgency is essentially a contest of attrition that lends itself to classical combat modeling [29], [30].

5.1 Confronting Entrenched Insurgents

Similarly to [3], Kaplan et al. [31] use modified Lanchester models to study the force allocation problem of both the government and the insurgents and, using a sequential force allocation game between the two sides, obtain an equilibrium. It is shown that the insurgents' optimal strategy depends on the government level of situational awareness; when the government has perfect intelligence, in equilibrium the insurgents concentrate their force in a single stronghold that the government either attacks or leave unengaged, depending upon the resulting casualty count. Otherwise, under reasonable assumptions regarding the government's behavior and intelligence capabilities, it is optimal for the insurgents to "spread out" in a way that maximizes the number of soldiers required to win all battles. This type of behavior was observed during the 2006 war against the Hezbollah in Lebanon, and the 2014 war against the Hamas in the Gaza Strip. On the other hand, for a given allocation of insurgents across strongholds, it is shown that an optimal selection of insurgents' strongholds to attack can be (approximately) accomplished with a simple knapsack rule that depends on the force size of the insurgents in a certain stronghold, the one-on-one fire-exchange relative strength, and the level of situational awareness.

5.2 Controlling Territories

Based also on a Lanchester setting, a model that accounts for the split between territorial regions loyal to the government and regions favoring the insurgents is described in [32]. Let B denote the government forces and R the insurgents. Also, the fraction of regions inhabited by government supporters is S and the fraction of insurgents' supporters country is C, $S + C = 1$ – see Figure 3. SB and CR indicate the fraction of regions that are liberated. That is, SB is the proportion of the S regions approvingly controlled by the government, and CR is the proportion of the C regions approvingly controlled by the insurgents. Similarly, SR and CB indicate subjugated regions – regions coercively controlled by the other side. Solid lines in Figure 3 indicate changes in control due to liberation, while dashed lines indicate subjugation. Regions do not change allegiances even under occupation, and each side can exert military force, to liberate or subjugate, only out of regions that support that side. It is shown that, contrary to classical Lanchesterian insights regarding traditional force-on-force engagements, the outcome of an insurgency is independent of the initial force sizes; it will only depend on the fraction of regions supporting each side and the combat effectiveness of each side. Moreover, unlike legacy force-on-force situations, counterinsurgency allows for stalemates (see also Section 3 above). Very often it is hard to avoid stalemate without foreign support. The foreign support in Libya in 2011 led to a victory by the insurgents, while lack of such support in Syria has resulted in an on-going stalemate where some regions are controlled by the Assad forces and others are controlled by the insurgents. The model's predictions have been consistent with the recently observed situations (2010-2014) in Afghanistan, Libya and Syria.

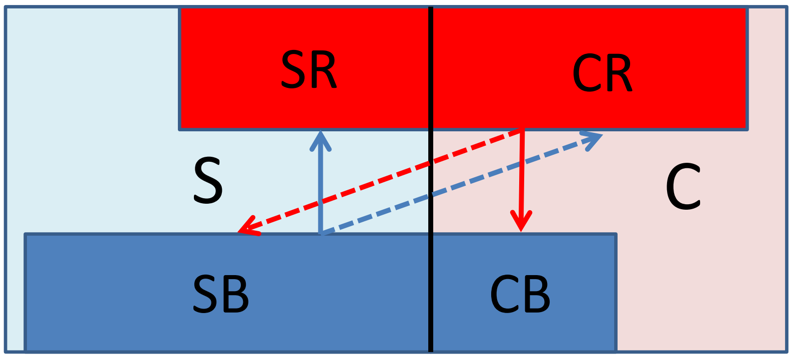

Figure 3: Schematic Dynamics of the Territorial Model

5.3 Tactical Issues

A common threat in insurgencies is road-side attacks by improvised explosive devices [33]. A model addressing this problem is given in [34]. Based on collected data on attacks against coalition forces in Iraq, the paper presents a stochastic, game-theoretic, model for optimally allocating clearing devices by the Coalition forces on a network of roads. The insurgents are strategic; they observe the Coalition forces' actions and they react to them. A related problem is the optimal interaction between a single military convoy of government forces and a single route-clearing team operating on a single roadway [35].

Another common threat by insurgencies – mostly used as terrorizing and coercing means against civilians – is suicide bombing. A person acts as a live bomb, killing and harming other people while killing himself or herself. The physics of this type of attack is studied in [36], where the effect of crowd blocking emerges as a significant factor in estimating the number of casualties. Mitigation tactics for this type of attack are examined, using probability models, in [37]. It is shown that even under best-case assumptions regarding the effectiveness and timeliness of widespread deployment of detection sensors against potential attackers, the expected number of casualties will not be significantly reduced compared to no detection. Thus, suicide-bomber-detector schemes will not likely to prove effective in protecting civilian populations from random pedestrian suicide-bomber attacks. The effort should be focused in prevention, rather than mitigation. In other words – invest in intelligence.

Summary

Insurgencies are different than regular armed conflicts in the way they are physically manifested and the role civilians play in them. These differences are reflected in mathematical models that attempt to study, analyze and understand this type of confrontations. From purely attritional focused models that characterize legacy force-on-force abstractions, insurgency models move to behavioral, social, political and economic domains. The main challenges in insurgency modeling lie in these domains and are mostly associated with data collection and interpretation. Insurgency modeling will experience a huge leap in application and relevance once effective and reliable data-collection methods will be in place for monitoring public mood, sentiments and behavior doing such armed conflicts. The advent and proliferation of observable social networks may give this capability a significant boost.

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